SS3011

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Space Technology and Application

Space Technology and Applications

"Orbitology"

Naval Postgraduate School
Monterey, California

Introduction to Orbital Mechanics:

- **S**tudy of motions of artificial satellites, space vehicles, or planetary bodies moving under the influence gravity, atmospheric drag, thrust, etc.

- **M**odern off shoot of *celestial mechanics* --study of motions of natural celestial bodies, sun, moon, planets.

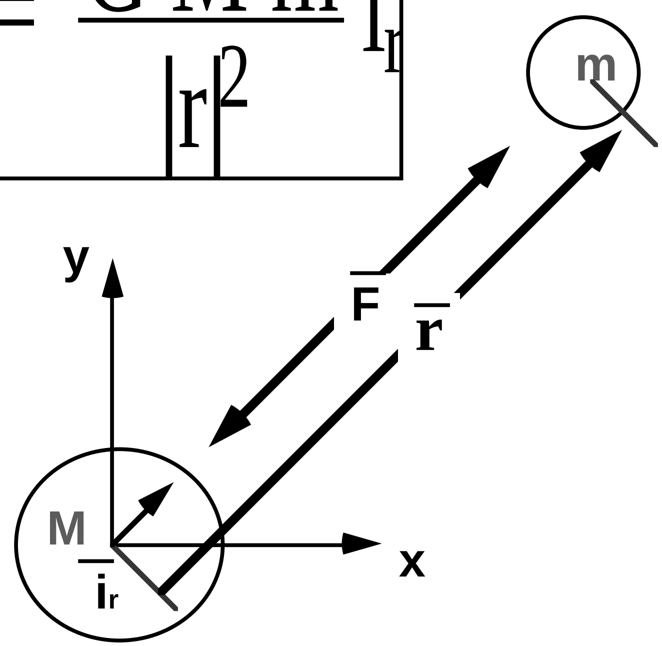
- **B**eyond the dissipating effects of Earth's atmosphere, the celestial motion of a body is mainly determined by *Gravitational Force*

Gravitational Physics

Gravitational Physics

- **Now a bit of *"gravitational physics"***

$$\boxed{\bar{F}_{Mm} = \frac{G\,M\,m}{|r|^2}\,\bar{i}_r}$$



"Inverse-square" law "potential" field

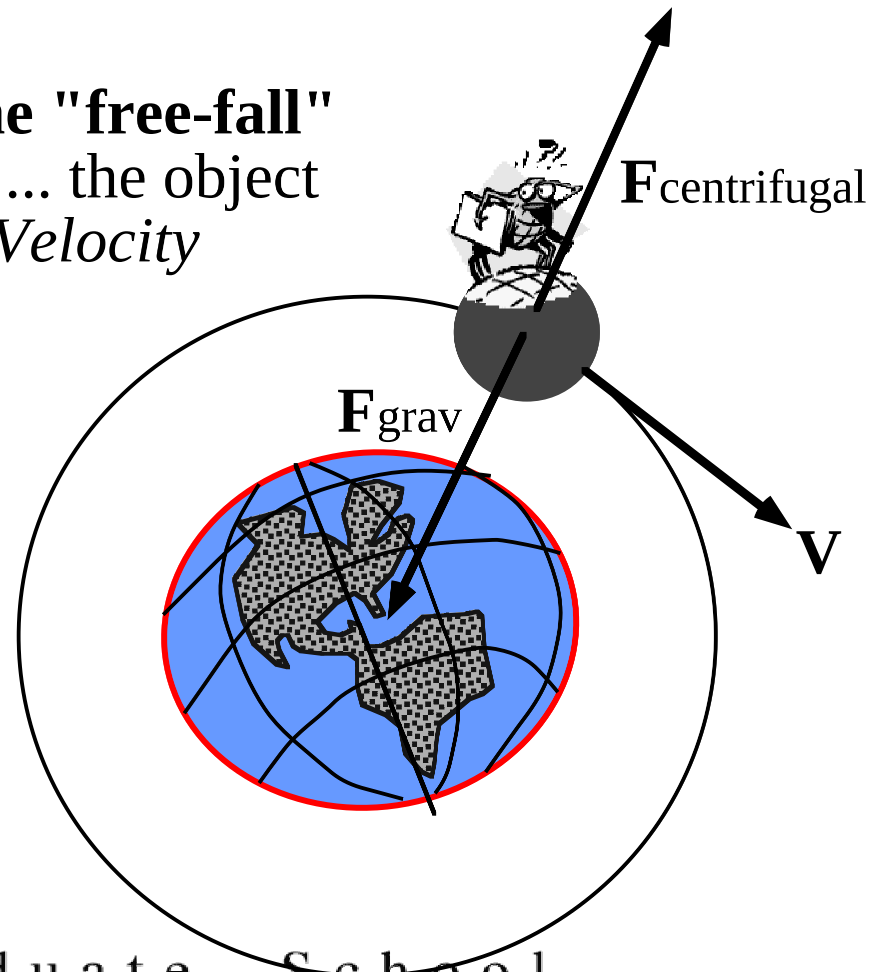
Isaac Newton, (1642-1727)

Orbital Velocity

Orbital Velocity

- **Object in orbit is actually in "free-fall"**
  that is ... the object is literally falling around the Earth (or Planet)

- **When the *Centrifugal Force* of the "free-fall"** counters the Gravitational Force ... the object is said to have achieved *Orbital Velocity*



Ingnoring Drag ... for a Circular orbit

$$\overline{F}_{grav} = \overline{F}_{centrifugal}$$

$$\Downarrow$$

$$\frac{G\,M\,m}{|r|^2} = m\,\omega^2\,|r| = m\left[\frac{V}{|r|}\right]^2 |r|$$

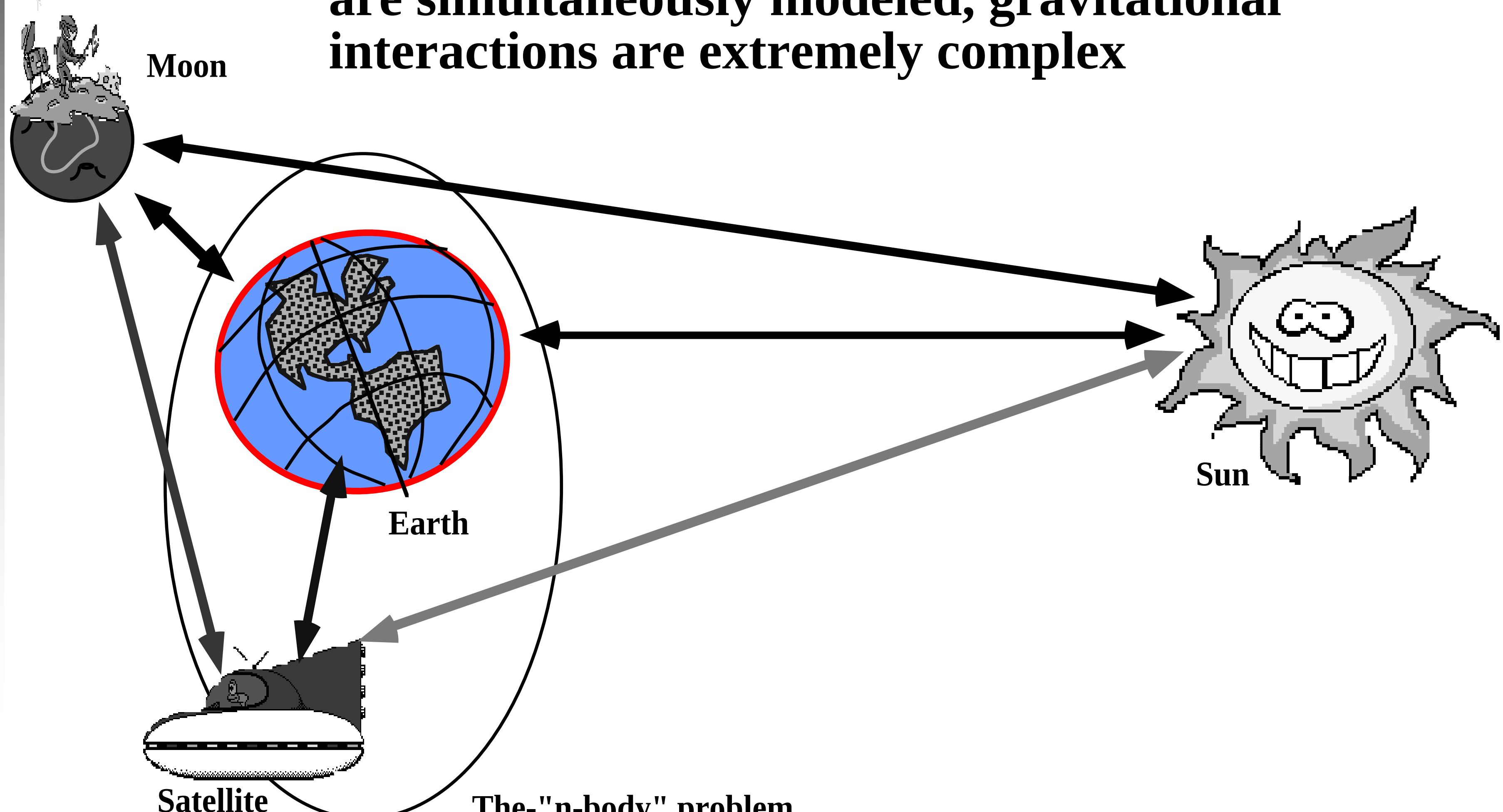
$$\Downarrow$$

$$V = \sqrt{\frac{G\,M}{|r|}}$$

The *"n-Body"* Problem

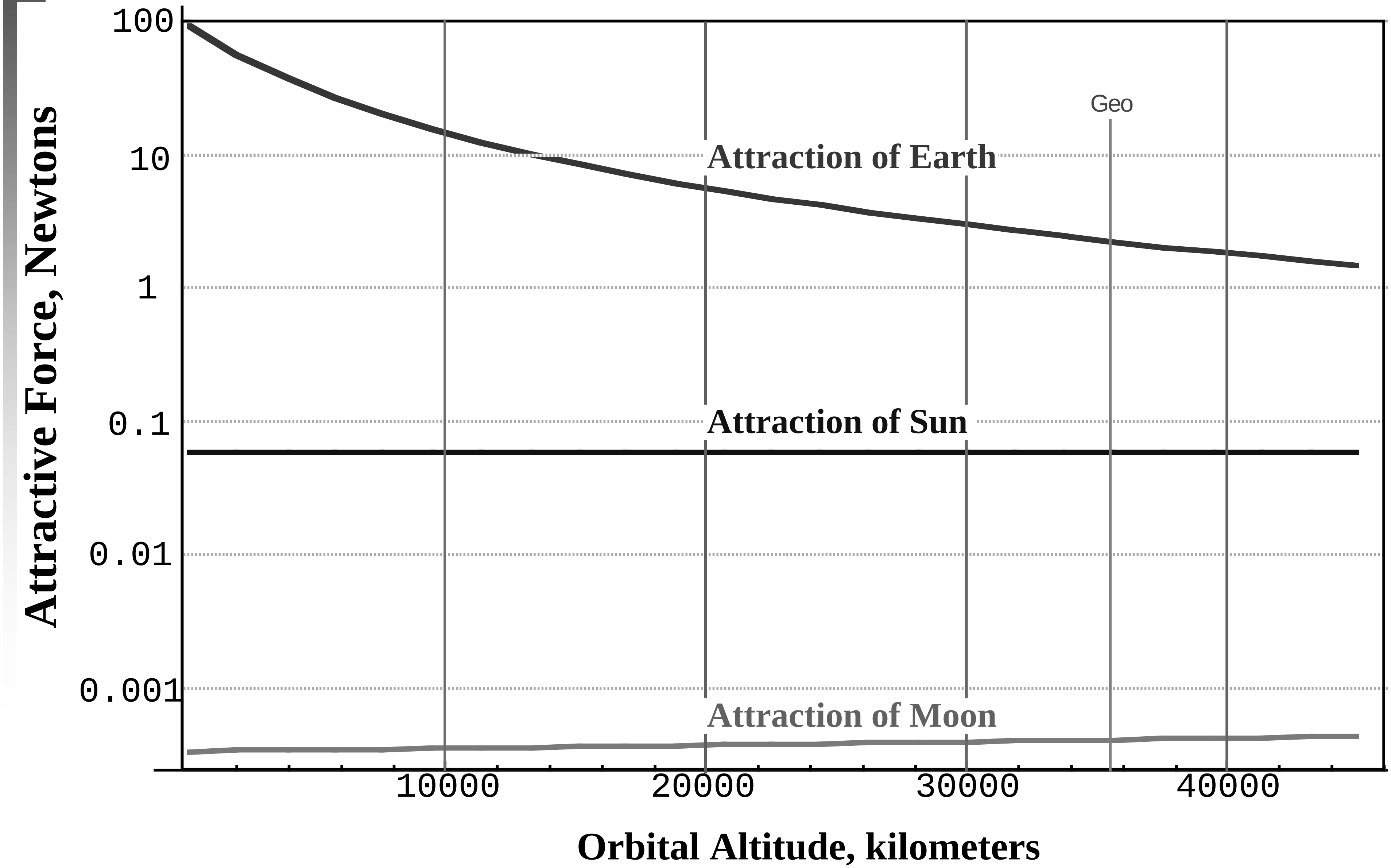
The "n-Body" Problem

- **If effects of Sun, Moon, earth, and Satellite mass are simultaneously modeled, gravitational interactions are extremely complex**



The-"n-body" problem

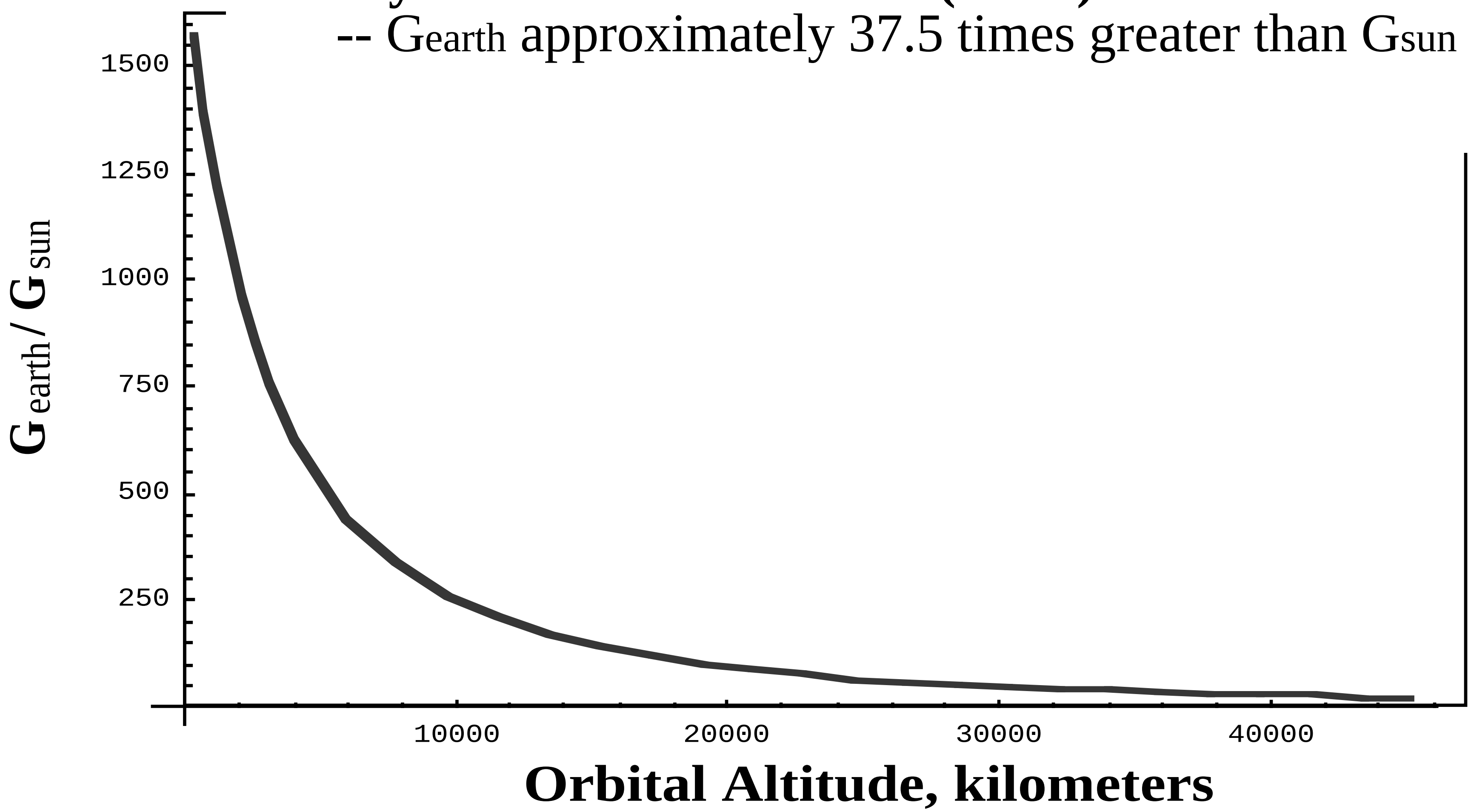
Gravitational Attraction on a 10,000 kg Spacecraft



Gravitational Attraction of Earth Relative to Sun

Gravitational Attraction of
Earth Relative to Sun

- **Low Earth Orbit (LEO)**
  -- G_{earth} approximately 2000 times greater than G_{sun}
- **Geosychronous Earth Orbit (GEO)**
  -- G_{earth} approximately 37.5 times greater than G_{sun}

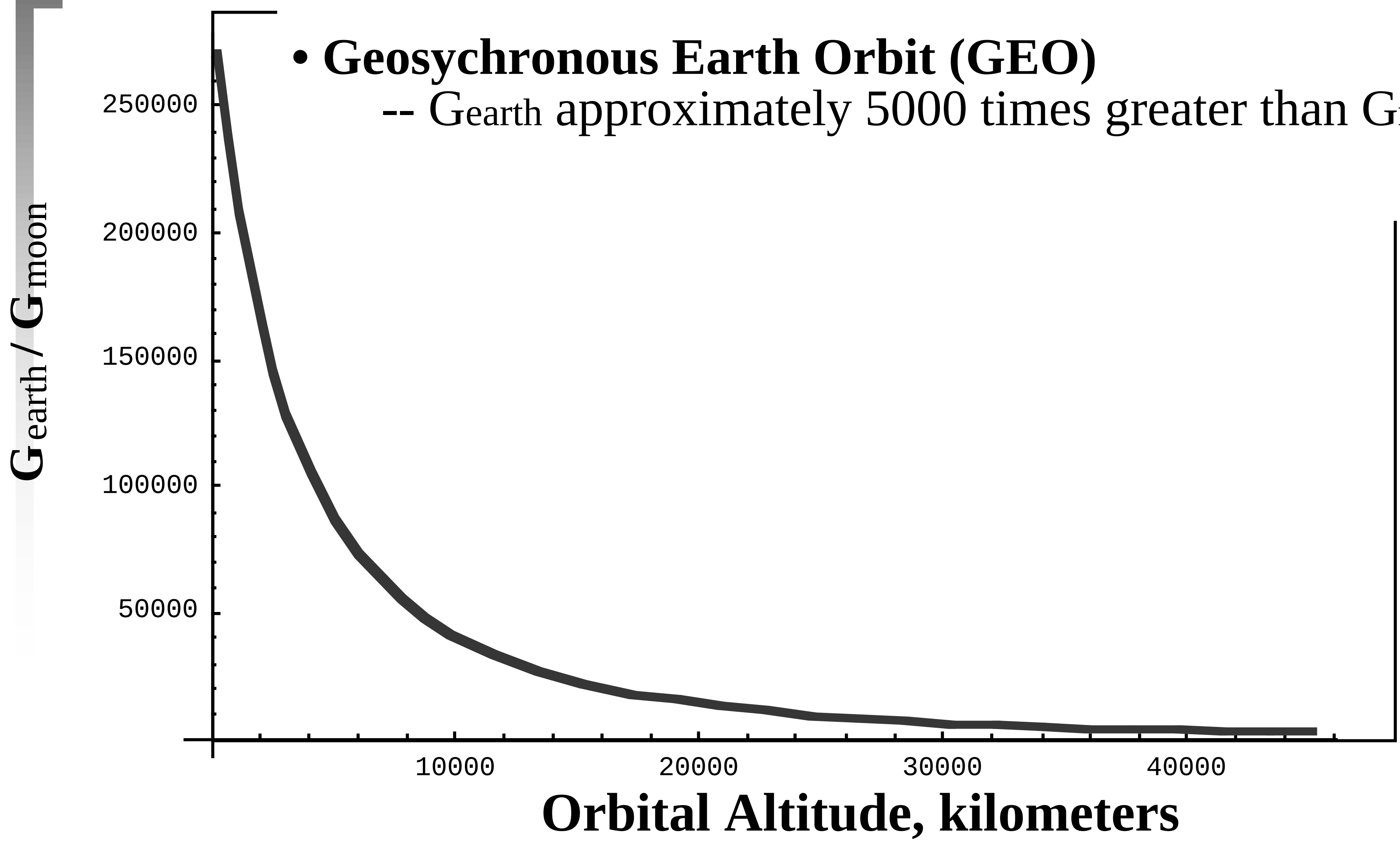


Gravitational Attraction of Earth Relative to Moon

Gravitational Attraction of Earth Relative to Moon

- **Low Earth Orbit (LEO)**
  -- G_{earth} approximately 27,000 times greater than G_{moon}

- **Geosychronous Earth Orbit (GEO)**
  -- G_{earth} approximately 5000 times greater than G_{moon}



The Two-Body Problem

The Two-Body Problem

- **For earth orbit, since the Earth's gravitational attraction is so much stronger than the Sun and Moon**

- **Can approximate most orbital dynamics by considering only the effects of the Earth on the satellite**
  (Clearly the effect of the satellite on the earth is negligible)

- **The-so called *two-body universe***

- **Gravitational attractions of sun and moon are considered as *perturbations* to the two-body problem**

- **In the *two-body universe* ... if the effect of drag ignored the motions of the satellite are exactly described by *Kepler's Laws***

Kepler's laws:

Kepler

• **R**oot of orbital mechanics traced back to laws of planetary motion for posed by Johannes Kepler, Imperial Mathematician to the Holy Roman Emperor, (1609 and 1619)

• **K***epler's laws* are a reasonable approximation of the dynamics of a small body orbiting around a much larger body in a 2-body universe

• **I**nteresting to note that Kepler derived his laws of planetary motion by *observation only*.

• **H**e did not have calculus available to assist him.*That* had to wait almost 100 years for *Sir Isaac Newton!*

Kepler's laws: (concluded)

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Kepler

• **Kepler's First Law:** *In a two body universe, orbit of a satellite is a conic section with the Earth centered at one of the focii*

• **Kepler's Second Law:** *In a two body universe, radius vector from the Earth to the satellite sweeps out equal areas in equal times*

• **Kepler's Third Law:** *In a two body universe, square of the period of any object revolving about the Earth is in the same ratio as the cube of its mean distance*

• ***Later*** *We'll Derive These Laws from First Principles Using Newton's Laws*

Kepler's First Law: Conic Sections:

Kepler's First Law:
Conic Sections:

- **4 Possible orbital paths:**

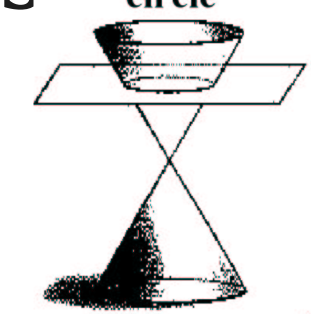
  Circle:

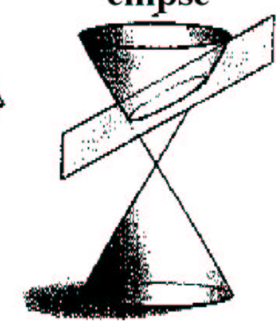
  Ellipse:

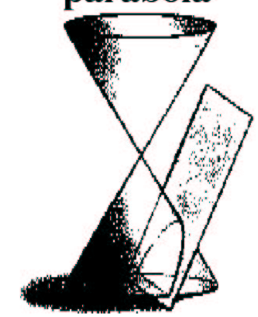
  Parabola:

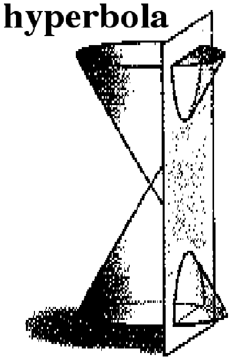
  Not in this course

  Hyperbola:









Circular Orbits:

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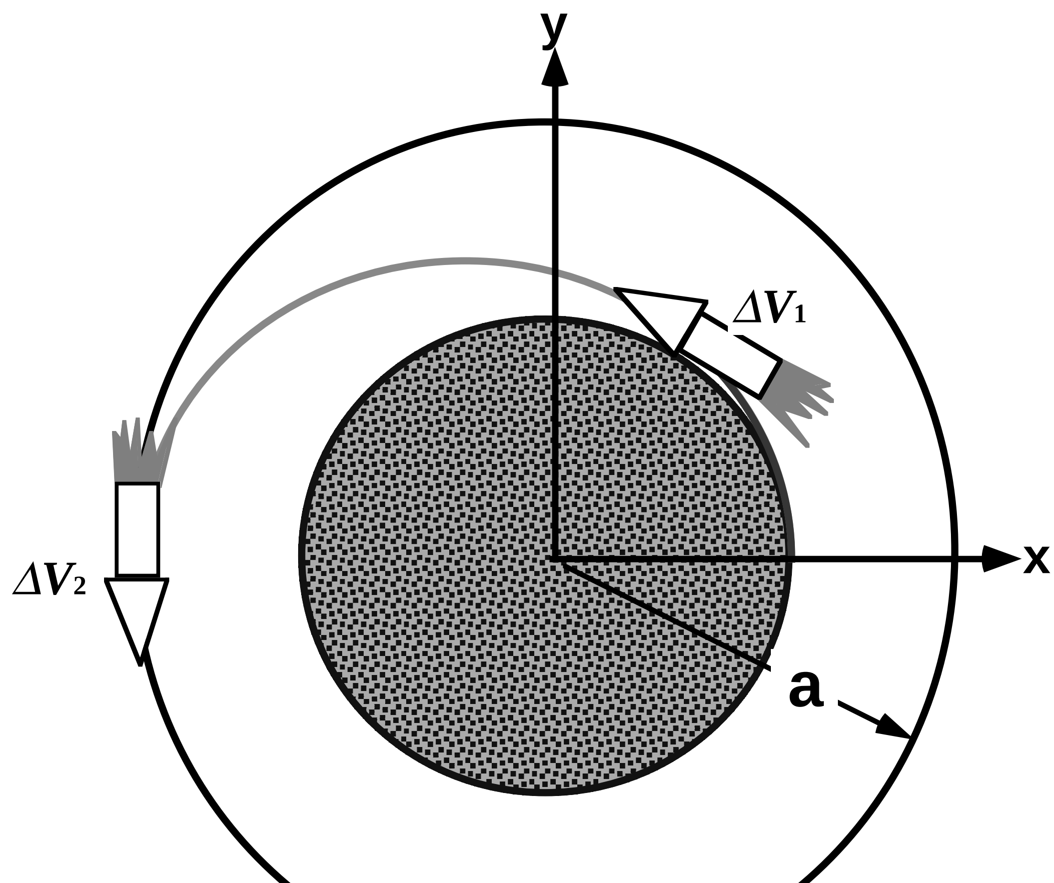


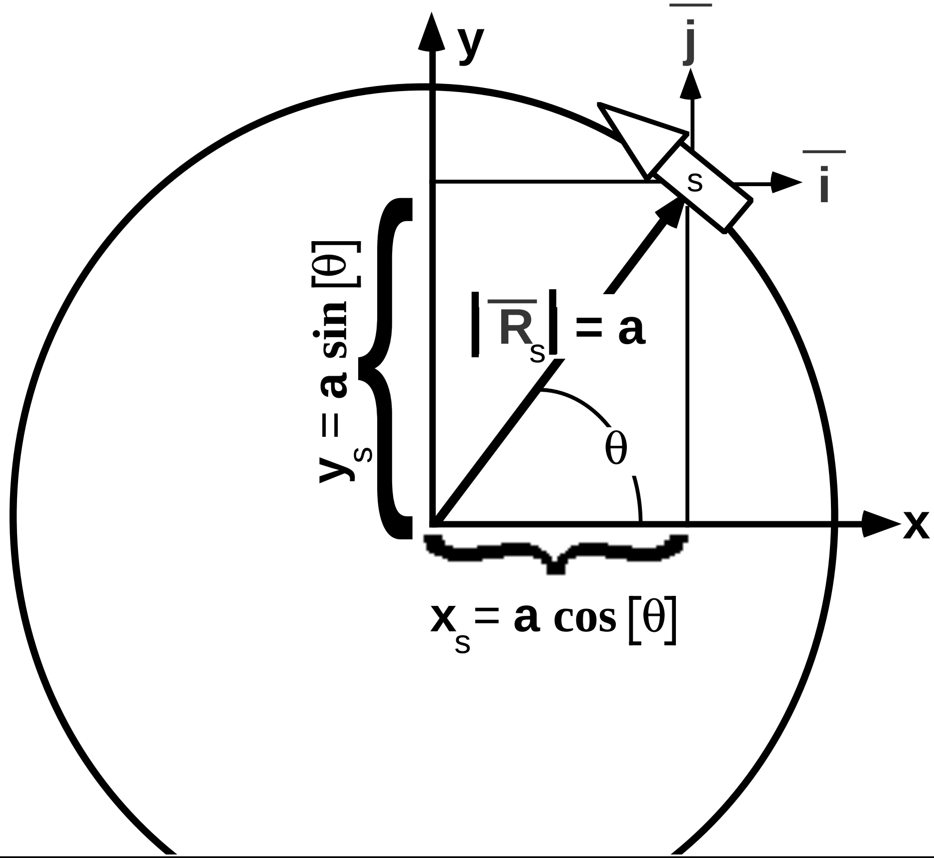
$$\left[\frac{x - x_c}{a}\right]^2 + \left[\frac{y - y_c}{a}\right]^2 = 1$$

for earth orbit

$x_c, y_c = \{0,0\}$

Circular Orbits: (cont;d)

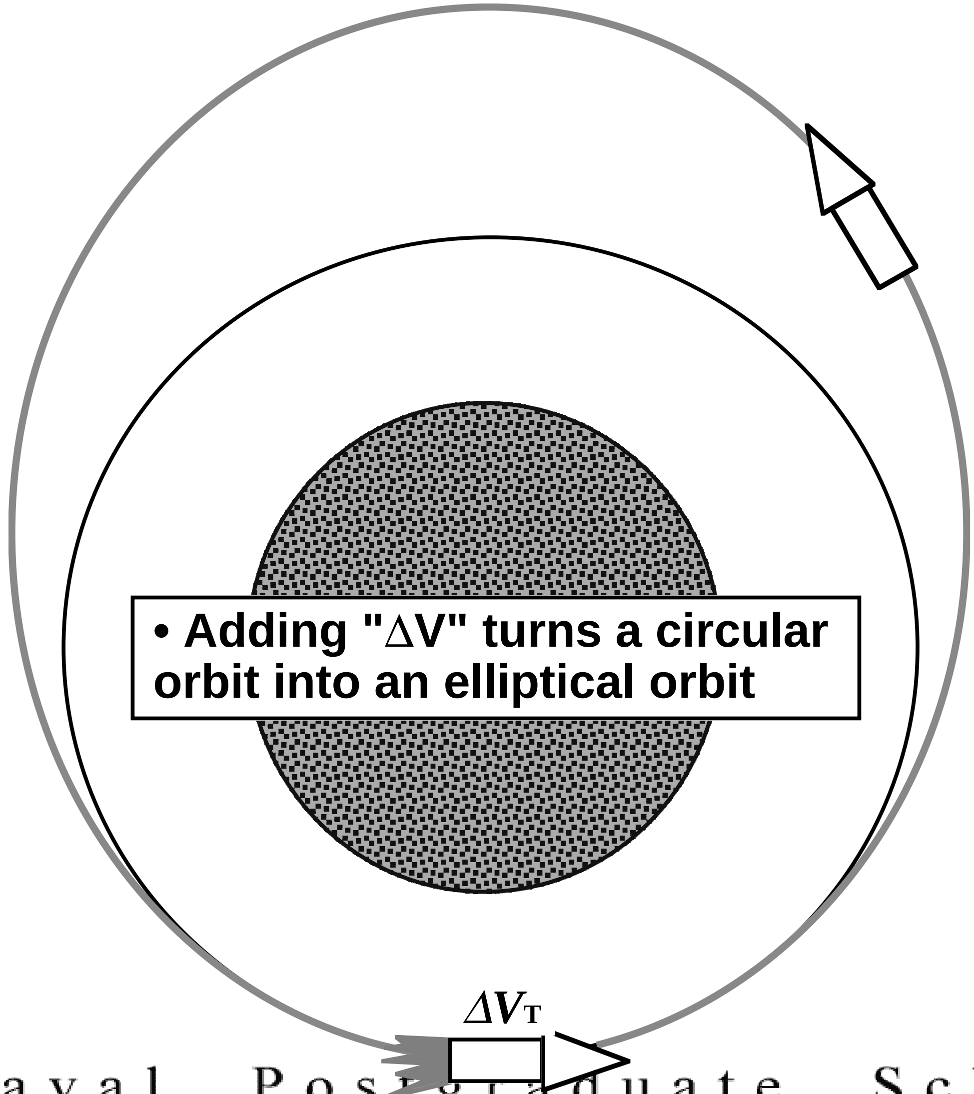
Circular Orbits:(cont;d)



$$\text{Position vector: } \bar{R}_s = x_s\,\bar{i} + y_s\,\bar{j} \equiv \begin{bmatrix} x_s \\ y_s \end{bmatrix}$$

Elliptical Orbits:

Elliptical Orbits



What is an Ellipse?

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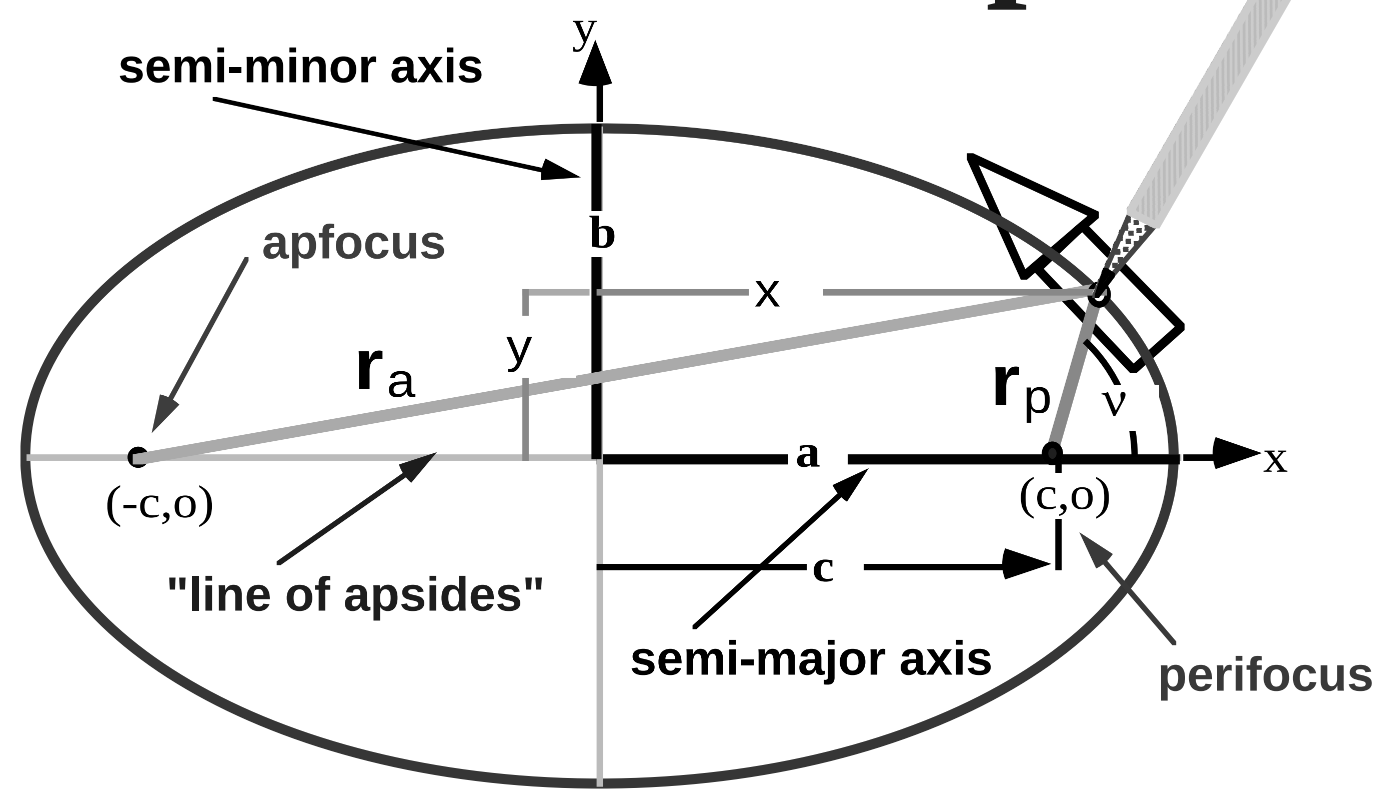


$$c = a\sqrt{1 - \left[\frac{b}{a}\right]^2}$$

Geometry of an Ellipse

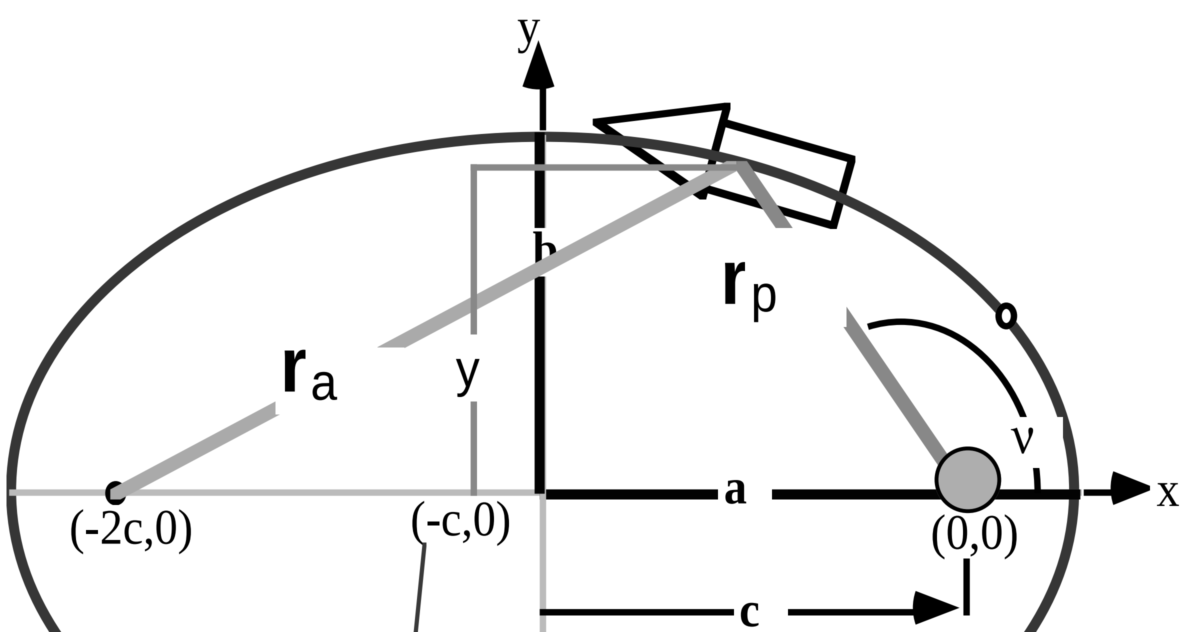
$$\boxed{|\mathbf{r}_a| + |\mathbf{r}_p| = 2\,\mathbf{a}}$$

The General Ellipse Equation

- **Cartesian non-dimensional form of the ellipse equation**

$$\frac{[x - x_c]^2}{a^2} + \frac{[y - y_c]^2}{b^2} = 1$$

Ellipse Equation: Earth Centered at origin



Earth (perifocus) centered at origin:

$$\frac{[x + c]^2}{a^2} + \frac{[y\ ]^2}{b^2} = 1$$

Polar-Form of the Ellipse Equation

Polar-Form of the Ellipse Equation

(concluded)

- **Defining the elliptical *eccentricity* as**

$$e \equiv \sqrt{1 - \frac{b^2}{a^2}}$$

- **The *polar form* of the *ellipse equation* reduces to**

$$\boxed{r_p = \frac{a\left[1 - e^2\right]}{\left[1 + e \cos(\nu)\right]}}$$

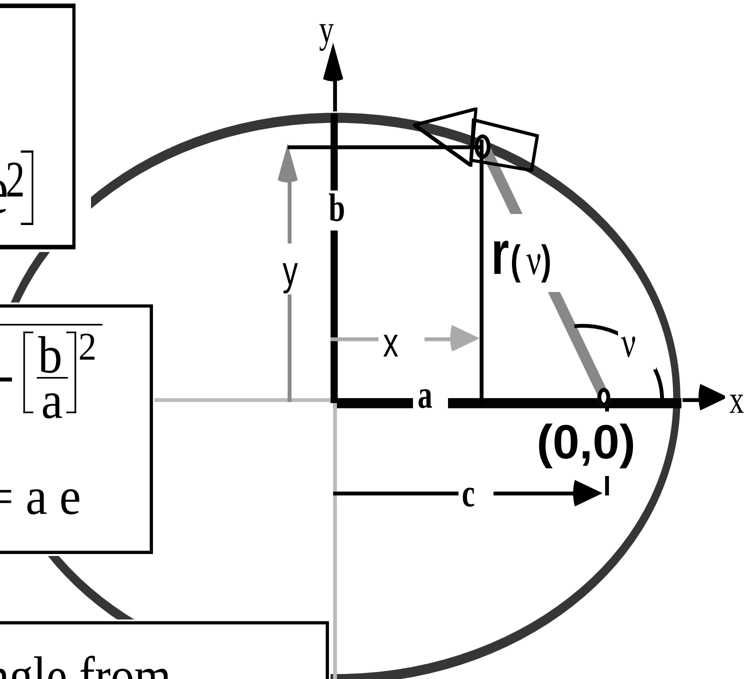
Parameters of the Elliptical Orbit

a: semi-major axis:

b: semi-minor axis: $b^2 = a^2\left[1 - e^2\right]$

e: orbital eccentricity $\Rightarrow e = \sqrt{1 - \left[\frac{b}{a}\right]^2}$

c: perifocus $\Rightarrow c = a\sqrt{1 - \left[\frac{b}{a}\right]^2} = a\,e$

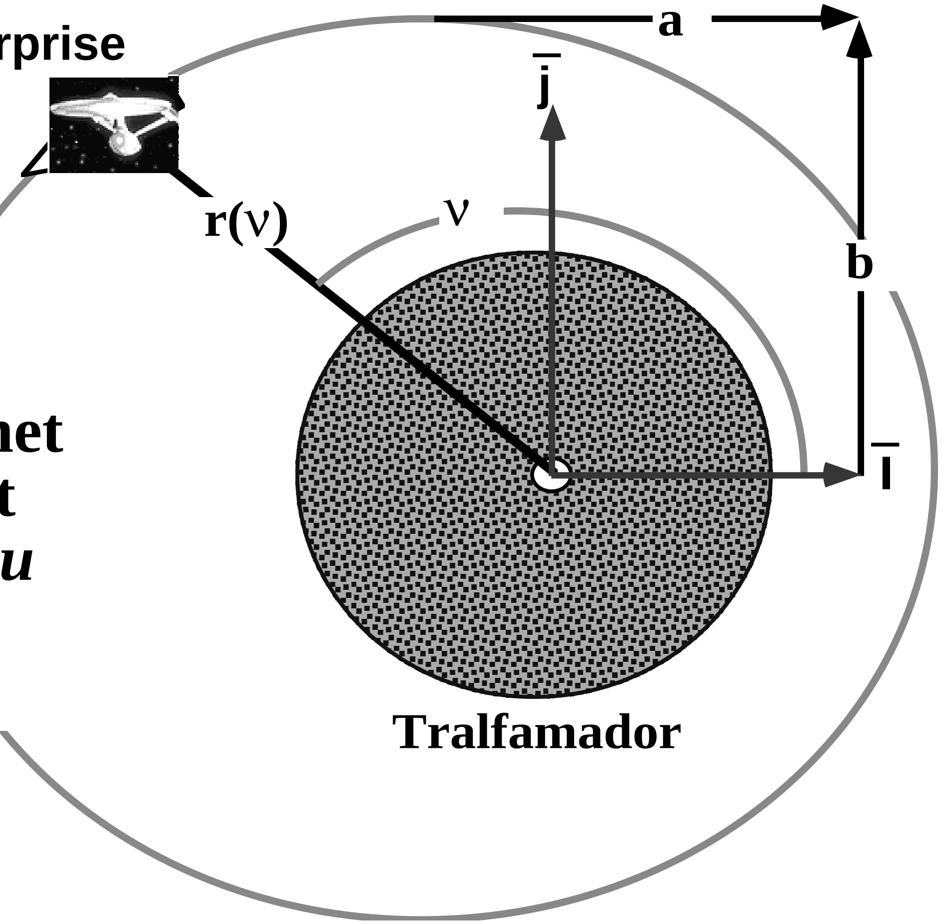


ν: true anomaly $\Rightarrow$ Angle from perapsis to satellite

(ν): orbital radius $\Rightarrow r(\nu) = \dfrac{a\left[1 - e^2\right]}{\left[1 + e\cos(\nu)\right]}$

Homework (elliptical orbits)

Read *Sellers* pp. 104-139



- **Starship Enterprise orbits planet *Tralfamador* in an elliptical orbit with semi-major axis *a = 0.02 Au* *(*astronomical units)* and semi-major axis *b = 0.01 Au***

Homework:Elliptical Orbits
(cont'd)

Homework: Elliptical Orbits

(cont'd)

- **Compute the peri*famador (Minimum distance)* and the ap*famador* (Maximum distance) of the orbit**

- **Show that** $$\frac{[r_{max} + r_{min}]}{2} = a$$

- **Show that** $$\frac{[r_{max} - r_{min}]}{[r_{max} + r_{min}]} = e$$

(do all calculations both symbolically and numerically)